Table 1. Parameter Values Used in Simulation

Number of variables	2
Period size	60
Level of significance, MTE	0.10
Level of significance, $T^2_{p,2N-2}$ test	0.11
Variances of measurement errors	1.0, 2.0
Number of simulation trials	10,000

(resp., power) for a steady state (resp., nonsteady state) simulation. It should be noted that the probability of type I error and power of the MTE must be evaluated through simulation. However, for the $T^2_{p,2N-2}$ test these performance characteristics can be obtained analytically as described in the Appendix of Narasimhan et al. (1986).

The results of the simulation are presented in Tables 1 and 2. In Table 1, the values of the parameters that are the same for all simulation cases are listed. The levels of significance are chosen such that the probability of type I error for both methods is the same. Therefore, both methods are compared on the same basis.

The different simulation runs and their results are given in Table 2. For each simulation run, the magnitudes of the relative changes in the true values for the variables (δ_i/q_{ii}) are listed in columns 2 and 3. The proportion of trials rejected by the MTE is given in column 4, and the power of the $T^2_{p,2N-2}$ test obtained theoretically is given in column 5. In simulation run 1, the changes in the true values of all variables are equal to zero. Hence, for this run the proportion of trials rejected is equal to the probability of type I error for the MTE. Note that this value is the same as the level of significance of the $T^2_{p,2N-2}$ test given in Table 1, and therefore the probability of type I error is the same for both methods. The proportion of trials rejected for the other simulation cases represents the power of the MTE.

The error in the estimates of probability of type I error and power is ± 0.01. Therefore, comparing columns 4 and 5 of Table 2, we can say that both methods give almost the same power. It should be noted that both methods give the same power when the relative changes in the variables are small (run 4) or large (run 2), or when only one variable undergoes a change in its true value (runs 5, 6 and 7). Similar results were obtained for the case of three variables, but these are not listed since they do not provide any new information. Based on these results, we can conclude that both methods give the same performance.

In some applications, it may also be important to know which variables caused a change in the state of the process to be detected. This is readily obtained in the case of the MTE by identifying the variables for which $m_i(NS)$ is greater than $m_i(S)$. In the case of $T^2_{p,2N-2}$ test, this information can be obtained only through additional testing. The MTE can also easily be modified to allow for prespecified tolerances for each variable within a steady state by including the tolerance in Eq. 3 when calculating the test statistic for each variable. However, one limitation of the MTE is that it is applicable only if the variables are independent. Another limitation is that the probability

Table 2. Simulation Results

Run No.	Relative Changes in Variables δ_1/q_{11}	δ_2/q_{22}	Proportion of Trials Rejected by MTE	Power of $T^2_{p,2N-2}$ Test
1	0.0	0.0	0.11	—
2	0.5	0.5	0.97	0.97
3	0.25	0.25	0.54	0.54
4	0.15	0.15	0.27	0.28
5	0.7	0.0	0.96	0.97
6	0.35	0.0	0.52	0.54
7	0.15	0.0	0.19	0.19

of type I error of the method is not readily given by the level of significance, as in the case of the $T^2_{p,2N-2}$ test. The user has to adjust iteratively the level of significance for the MTE in order to obtain a desired probability of type I error.

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Notation

$m_i(S)$ = belief for steady state proposition based on variable i
$m_i(NS)$ = belief for nonsteady state proposition based on variable i
$m_i(U)$ = belief for uncertainty based on variable i
N = number of measurements in each period
p = number of variables
$Pr\{.\}$ = probability of .
$\underline{Q}$ = covariance matrix of measurement errors
q_{ii} = ith diagonal element of $\underline{Q}$
$t^2_{1,i}$ = T^2 statistic for variable $\bar{i}$, Eq. 3
$s_{k,i}$ = sample variance of variable i in period $\underline{k}$
$T^2_{a,b}$ = T^2 random variable with numerator degrees of freedom a and denominator degrees of freedom b
$T^2(\alpha)$ = upper T^2 percentile
v_{kj} = jth vector of random measurement errors in period k
$\underline{x}_{kj}$ = jth measurement vector in period k
$x_{kj,i}$ = jth measurement of variable i in period $\underline{k}$
x_k = vector of true values of variables in period k
$\bar{x}_{k,i}$ = mean value of measurements for variable i in period k

Greek letters

α = level of significance
Π_i = product over all i
δ_i = value change for variable i
Σ_i = sum over all i

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Errata

In the paper titled "Generalized Likelihood Ratio Method for Gross Error Identification" by S. Narasimhan and R. S. H. Mah (Sept. 1987), the Grant No. on p. 1520 should read CPE 8519182, not CPE 811 5161.